Special type of series

Question 1. Sum of series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \text{----------}\infty$

Solution : This is arithmetico-geometric series

First term (a) = 1, common difference (d) = 3, common ratio (r) = ⅕

To solve this we multiply both sides by common ratio (r) = ⅕

$$S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \text{----------}\infty$$

$$\frac{S}{5} = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \text{----------}\infty$$

$$S - \frac{S}{5} = 1 + \left(\frac{4}{5} - \frac{1}{5}\right) + \left(\frac{7}{5^2} - \frac{4}{5^2}\right) + \left(\frac{10}{5^3} - \frac{7}{5^3}\right) + \text{---------------}\infty$$

$$\frac{4S}{5} = 1 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \text{----------}\infty$$

$$\frac{4S}{5} = 1 + \frac{3}{5}\left(1 + \frac{1}{5} + \frac{1}{5^2} + \text{----------}\infty\right)$$

$$\frac{4S}{5} = 1 + \frac{3}{5}\left(\frac{1}{1-1/5}\right)$$

$$\frac{4S}{5} = 1 + \frac{3}{5} \text{ x } \frac{5}{4}$$

$$\frac{4S}{5} = \frac{7}{4}$$

$$S = \frac{35}{16}$$

Or (Short cut trick)

This is arithmetico-geometric series upto infinity so we apply formula $S\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

a = first term, d = common difference, r = common ratio

a = 1, d = 3, r = 1/5

$$S_\infty = \frac{1}{1-1/5} + \frac{(3).(1/5)}{(1-1/5)^2} = \frac{5}{4} + \frac{(3/5)}{(16/25)} = \frac{5}{4} + \frac{15}{16} = \frac{35}{16}.$$

Question 2. **Sum of series** $1 + \frac{7}{2} + \frac{13}{2^2} + \frac{19}{2^3} + \text{----------}\infty$

Solution : This is arithmetico-geometric series

First term (a) = 1, common difference (d) = 6, common ratio (r) = ½

To solve this we multiply both sides by common ratio (r) = ½

$$S = 1 + \frac{7}{2} + \frac{13}{2^2} + \frac{19}{2^3} + \text{----------}\infty$$

$$\frac{S}{2} = \quad \frac{1}{2} + \frac{7}{2^2} + \frac{13}{2^3} + \text{----------}\infty$$

$$S - \frac{S}{2} = 1 + \left(\frac{7}{2} - \frac{1}{2}\right) + \left(\frac{13}{2^2} - \frac{7}{2^2}\right) + \left(\frac{19}{2^3} - \frac{13}{2^3}\right) + \text{----------------}\infty$$

$$\frac{S}{2} = 1 + \frac{6}{2} + \frac{6}{2^2} + \frac{6}{2^3} + \text{----------}\infty$$

$$\frac{S}{2} = 1 + 3\left(1 + \frac{1}{2} + \frac{1}{2^2} + \text{----------}\infty\right)$$

$$\frac{S}{2} = 1 + 3\left(\frac{1}{1-1/2}\right)$$

$$\frac{S}{2} = 1 + 3 \times 2$$

$$\frac{S}{2} = 7$$

S = 14

Or (Short cut trick)

This is arithmetico-geometric series upto infinity so we apply formula $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

a = first term, d = common difference, r = common ratio

a = 1, d = 6, r = 1/2

$S_\infty = \frac{1}{1-1/2} + \frac{(6).(1/2)}{(1-1/2)^2} = 2 + \frac{3}{1/4} = 14$.

Question 3. Find the sum of $10^3 + 11^3 + 12^3 + 13^3 + \text{--------} + 22^3$

Solution : In this we can find $S = \sum_{n=1}^{22} n^3 - \sum_{n=1}^{9} n^3 = \{\frac{22(22+1)}{2}\}^2 - \{\frac{9(9+1)}{2}\}^2$

$= (11 \times 23)^2 - (45)^2 = 64{,}009 - 2025 = 61{,}984$

Question 4. Find the sum of $6^3 + 7^3 + 8^3 + 9^3 + \text{--------} + 20^3$

Solution : In this we can find $S = \sum_{n=1}^{20} n^3 - \sum_{n=1}^{5} n^3 = \{\frac{20(20+1)}{2}\}^2 - \{\frac{5(5+1)}{2}\}^2$

$= (210)^2 - (15)^2 = 44{,}100 - 225 = 43{,}875$