Sum of Infinite Terms

Sum of an Infinite Geometric Series

If $a_1 + a_1 r + a_1 r^2 + \cdots$ is an infinite geometric series, with $|r| < 1$, then the sum S of all of the terms of this series is given by

$$S = \frac{a_1}{1 - r}.$$

a = first term,   r = common ratio

Question 1. **Find the sum of an infinite G.P. Whose first term is 28 and second term is 4 .**

Solution: **As per question ,**

$a = t_1 = 28$

$t_1 = ar = 4$

r(common ratio) = $\frac{t_2}{t_1} = \frac{ar}{a} = \frac{4}{28} = \frac{1}{7}$

$S_\infty = \frac{a}{1 - r}$

$S_\infty = \frac{28}{1 - \frac{1}{7}} = \frac{28 \times 7}{6} = \frac{98}{3}$

Question 2. **Evaluate** $11^{1/2} \times 11^{1/4} \times 11^{1/8} \times$ ----------------- ∞ .

Solution: $= 11^{1/2} \times 11^{1/4} \times 11^{1/8} \times$ ----------------- ∞ **infinite**

$= 11^{1/2+1/4+1/8}$ ----------------- ∞

Now, find the sum of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} +$ ------------ ∞

$S_\infty = \frac{a}{1 - r} = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$

So, $11^1 = 11$

Question 3. Find the sum to infinity of the given series: $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} +$ ------------∞

Solution: as we know that $S\infty = \frac{a}{1-r}$

$= \frac{1}{2}\{1 + \frac{1}{4} + \frac{1}{16} +$ ------------$\infty\}$

$a = 1$, r (common ratio) $= \frac{1}{4}$

$S\infty = \frac{a}{1-r} = \frac{1}{2}\{\frac{1}{1-\frac{1}{4}}\} = \frac{2}{3}$

Question 4. Find the sum of $0.\overline{7}$

Solution: We can write

$0.\overline{7} = 0.77777777$------------$\infty$(Repeating but not terminating) So we can apply the concept of infinite series to sum .

$0.\overline{7} = 0.77777777$

$= 0.7 + 0.07 + 0.007 +$ ------------∞

$= \frac{7}{10} + \frac{7}{10^2} + \frac{7}{10^3}$ ------------∞

$S\infty = \frac{a}{1-r} = \frac{\frac{7}{10}}{1-(\frac{1}{10})}$

$= \frac{(\frac{7}{10})}{(\frac{9}{10})} = \frac{7}{9}$

Or

If we convert $0.\overline{7}$ into a rational number $0.\overline{7} = \frac{7}{9}$ This will be equal to the sum of the series.

Question 5. If $x = 1 + \frac{1}{3} + \frac{1}{3^2} + \text{-------------}\infty$

$y = 1 + \frac{1}{4} + \frac{1}{4^2} + \text{-------------}\infty$, find the product of xy.

Solution : we find the sum of x series $S\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

Now we find the sum of y series $S\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$

$xy = \frac{3}{2} \times \frac{4}{3} = 2$

Question 6. If $x = 1 + \frac{1}{2} + \frac{1}{2^2} + \text{-------------}\infty$

$y = 1 + \frac{1}{6} + \frac{1}{6^2} + \text{-------------}\infty$, find the product of xy.

Solution : we find the sum of x series $S\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$

Now we find the sum of y series $S\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{6}} = \frac{6}{5}$

$xy = 2 \times \frac{6}{5} = \frac{12}{5}$