Very Important Theorem

Theorem : **The number of permutations of n things taken all at a time, when p things are similar of one kind, q things are similar of another kind, and so on and rest are different is:** $\frac{n!}{p!\ x\ q!}$

Question 1: **There are 4 red, 3 green and 3 blue stones. How many possible arrangements can be made by arranging all of them horizontally.**

Solution: **In this n = p + q + r = 4 + 3 + 3 = 10**

This is very simple problem based on the above theorem

$\frac{n!}{p!\ x\ q!\ x\ r!} = \frac{10!}{4!\ x\ 3!\ x\ 3!}$ **= 4200 ways.**

Question 2: **There are 6 orange, 4 pink and 5 red balls. How many possible arrangements can be made by arranging all of them vertically.**

Solution: **In this n = p + q + r = 6 + 4 + 5 = 15**

This is very simple problem based on the above theorem

$\frac{n!}{p!\ x\ q!\ x\ r!} = \frac{15!}{6!\ x\ 4!\ x5!}$ **= 630630 ways.**

Question 3: **How many possible arrangements are possible with the letters of the word BOOK.**

Solution: In this letter O is 2 times so possible arrangements are = $\frac{n!}{p!} = \frac{4!}{2!}$ =12 ways.

Question 4: How many possible arrangements are possible with the letters of the word Statistics.

Solution: In this letter S is 3 times, t is 3 times, i is 2 times, a is one time and c is one time so possible arrangements are

$$= \frac{n!}{p! x\, q!\, x\, r!} = \frac{10!}{3!\, x\, 3!\, x\, 2!} = \text{50,400 ways.}$$

Question 5: How many different numbers of seven digits divisible by 10 can be formed by using the digits 3, 2, 0, 3, 2, 2, 5 ?

Solution : In this case digits are 3, 2, 0, 3, 2, 2, 5.

The numbers which are divisible by 10 must have digit "0" at unit's place, in remaining digits two 3's, three 2's.

So, as per theorem the total developed numbers are = $\frac{n!}{p! x\, q!}$ $\frac{6!}{3!\, x\, 2!} = 60$.

Or

Let there are seven places to fill the seven digits □ □ □ □ □ □ □, unit's place can be filled by the digit "0" there is one way to fill unit place. Similarly ten's place can be filled by 6 ways, hundreds place can be filled by 5 ways and so on:

Total developed numbers are = $\frac{6 \, x \, 5 \, x \, 4 \, x \, 3 \, x \, 2 \, x \, 1 \, x \, 1}{3! x \, 2!}$ **= 60.**

Digits 2 and 3 are repeated by three times and two times respectively so as per theorem it is divided by 2! and 3!.